# SOLUTION TO PROBLEM #638
# PROPOSED BY MANJUL BHARGAVA

Tewodros Amdeberhan

DeVry Institute, Mathematics
630 US Highway One, North Brunswick, NJ 08902
amdberhan@admin.nj.devry.edu

**Problem #638:** Evaluate

$$\sum \frac{1}{\binom{n}{k}}, \tag{1}$$

where the summation ranges over all positive integers $n, k$ with $1 < k < n-1$.

**Solution:** First, we rewrite the sum in (1) as:

$$\sum \frac{1}{\binom{n}{k}} = \sum_{n=4}^{\infty}\sum_{k=2}^{n-2} \frac{1}{\binom{n}{k}} = \sum_{m=2}^{\infty}\sum_{k=2}^{\infty} \frac{1}{\binom{m+k}{k}}. \tag{2}$$

Next, we recognize that the inner sum in the last equality amounts to $\frac{2}{(m-1)(m+1)}$. This follows from (setting $a=0$)

$$\sum_{k=0}^{\infty} \frac{\binom{k}{a}}{\binom{m+k}{k}} = \frac{m(m-a-2)!a!}{(m-1)!} \tag{3}$$

and subtracting off the first two terms, whereas the last identity is easily provable (example, by the WZ method). Finally,

$$\sum_{m=2}^{\infty}\sum_{k=2}^{\infty} \frac{1}{\binom{m+k}{k}} = \sum_{m=2}^{\infty} \frac{2}{(m-1)(m+1)} = \frac{3}{2},$$

where the last equality is a consequence of telescoping. □

**References:**

[**P**] P #638, *The College Mathematics Journal,* **(29) #5,** November 1998.