SOLUTION TO PROBLEM #678
PROPOSED BY D. ATKINSON

[P] For $n = 0, 1, \ldots$, find the value of the double sum $\sum_{i=0}^{n} \sum_{j=0}^{n-i} \frac{(-1)^j}{i!j!}$ as a function of n.

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The result is achieved by a change in summation, namely summing *diagonally.*

$$\sum_{i=0}^{n} \sum_{j=0}^{n-i} \frac{(-1)^j}{i!j!} = \sum_{k=0}^{n} \sum_{i,j\geq 0}^{i+j=k} \frac{(-1)^j}{i!j!} = \sum_{k=0}^{n} \sum_{i=0}^{k} \frac{(-1)^{k-i}}{i!(k-i)!} = \sum_{k=0}^{n} \frac{(-1)^k}{k!} \sum_{i=0}^{k} (-1)^i \binom{k}{i}.$$

The well-known binomial identity $\sum_{i=0}^{k} (-1)^i \binom{k}{i} = \delta_0(k)$ implies that

$$\sum_{i=0}^{n} \sum_{j=0}^{n-i} \frac{(-1)^j}{i!j!} = \sum_{k=0}^{n} \frac{(-1)^k}{k!} \delta_0(k) \equiv 1, \forall n \geq 0. \qquad \square$$

References:

[**P**] P #678, *The College Mathematics Journal,* **(31) #3,** May 2000.

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