PROOF OF FORMULA 3.193

$$\int_0^n x^{\nu-1}(n-x)^n\,dx = \frac{n^{\nu+n}\,n!}{(\nu)_{n+1}} = \frac{n^{\nu+n}\,n!}{\nu(\nu+1)\cdots(\nu+n)}$$

Parameter restrictions. Convergence of the integral near $x=0$ requires $\operatorname{Re}\nu > 0$ and near $x=n$ it requires $\operatorname{Re} n > -1$.

Evaluation. The change of variables $x = nt$ produces

$$\int_0^n x^{\nu-1}(n-x)^n\,dx = n^{\nu+n}\int_0^1 t^{\nu-1}(1-t)^n\,dt.$$

The integral representation (that appears as entry **3.191.3**)

$$B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1}\,dt$$

shows that the required integral is

$$\int_0^n x^{\nu-1}(n-x)^n\,dx = n^{\nu+n}B(\nu, n+1).$$

This is simplified to

$$B(\nu, n+1) = \frac{\Gamma(\nu)\Gamma(n+1)}{\Gamma(n+1+\nu)},$$

and the result follows from the relation

$$(a)_m = \frac{\Gamma(a+m)}{\Gamma(a)},$$

among the gamma function and the Pochhammer symbol

$$(a)_m = a(a+1)\cdots(a+m-1).$$

Scaling. This entry is simply a scaled version of **3.191.3**, so it should be eliminated.