FORMULA 3.221.2

$$\int_{-\infty}^{a} \frac{(a-x)^{p-1}}{x-b}\, dx = -\frac{\pi (b-a)^{p-1}}{\sin \pi p}$$

Let $y = a - x$ to obtain

$$\int_{-\infty}^{a} \frac{(a-x)^{p-1}}{x-b}\, dx = -\int_{0}^{\infty} \frac{y^{p-1}\, dy}{(b-a)+y}.$$

The change of variables $y = (b-a)t$ gives

$$\int_{-\infty}^{a} \frac{(a-x)^{p-1}}{x-b}\, dx = -(b-a)^{p-1} \int_{0}^{\infty} \frac{t^{p-1}\, dt}{1+t}.$$

The representation

$$B(u,v) = \int_{0}^{\infty} \frac{t^{u-1}\, dt}{(1+t)^{u+v}},$$

shows that the previous integral is $B(p, 1-p) = \pi / \sin \pi p$.