PROOF OF FORMULA 3.311.11

$$\int_0^\infty \frac{e^{px}-e^{qx}}{e^{rx}-e^{sx}}\,dx = \frac{1}{r-s}\left[\psi\left(\frac{r-q}{r-s}\right)-\psi\left(\frac{r-p}{r-s}\right)\right]$$

Write the integral as

$$\int_0^\infty \frac{e^{px}-e^{qx}}{e^{rx}-e^{sx}}\,dx = \int_0^\infty \frac{e^{-(r-p)x}-e^{-(r-q)x}}{1-e^{-(r-s)x}}\,dx.$$

The change of variables $t=(r-s)x$ gives

$$\int_0^\infty \frac{e^{px}-e^{qx}}{e^{rx}-e^{sx}}\,dx = \frac{1}{r-s}\int_0^\infty \frac{e^{-(r-p)t/(r-s)}-e^{-(r-q)t/(r-s)}}{1-e^{-t}}\,dt.$$

Entry **3.311.7** now gives the result.