PROOF OF FORMULA 3.311.13

$$\int_0^\infty \frac{e^{-px} + e^{-qx}}{1 + e^{-(p+q)x}} \, dx = \frac{\pi}{p+q} \operatorname{cosec} \left(\frac{\pi p}{p+q} \right)$$

Let $t = e^{-(p+q)x}$ to get

$$\int_0^\infty \frac{e^{-px} + e^{-qx}}{1 + e^{-(p+q)x}} \, dx = \frac{1}{p+q} \int_0^1 \frac{t^{r-1} + t^{-r}}{1+t} \, dt,$$

with $r = p/(p+q)$.

Entry **3.231.2** gives $\pi \operatorname{cosec} \pi r$ as the value of this last integral.