

PROOF OF FORMULA 3.311.8

$$\int_{-\infty}^{\infty} \frac{e^{-\mu x} dx}{b - e^{-x}} = \frac{\pi \cot(\pi\mu)}{b^{1-\mu}}$$

The  $t = e^{-x}$  to obtain

$$\int_{-\infty}^{\infty} \frac{e^{-\mu x} dx}{b - e^{-x}} = \int_0^{\infty} \frac{t^{\mu-1} dt}{b - t}.$$

The change of variables  $t = bs$  now gives

$$\int_{-\infty}^{\infty} \frac{e^{-\mu x} dx}{b - e^{-x}} = b^{\mu-1} \int_0^{\infty} \frac{s^{\mu-1} ds}{1 - s}.$$

To evaluate this integral, write

$$\int_0^{\infty} \frac{s^{\mu-1} ds}{1 - s} = \int_0^1 \frac{s^{\mu-1} ds}{1 - s} + \int_1^{\infty} \frac{s^{\mu-1} ds}{1 - s}.$$

The change of variables  $s \mapsto 1/s$  in the second integral yields

$$\int_0^{\infty} \frac{s^{\mu-1} ds}{1 - s} = \int_0^1 \frac{s^{\mu-1} - s^{-\mu}}{1 - s} ds.$$

This is evaluated in **3.231.1** as  $\pi \cot(\pi\mu)$ .