PROOF OF FORMULA 3.311.9

$$\int_0^\infty \frac{e^{-\mu x}\,dx}{b+e^{-x}} = \frac{\pi b^{\mu-1}}{\sin \pi\mu}$$

Let $t = e^{-x}$ to obtain

$$\int_0^\infty \frac{e^{-\mu x}\,dx}{b+e^{-x}} = \int_0^\infty \frac{t^{\mu-1}\,dt}{b+t}.$$

The change of variables $t = bs$ gives

$$\int_0^\infty \frac{e^{-\mu x}\,dx}{b+e^{-x}} = b^{\mu-1}\int_0^\infty \frac{s^{\mu-1}\,ds}{1+s}.$$

The integral representation

$$B(u,v) = \int_0^\infty \frac{t^{u-1}\,dt}{(1+t)^{u+v}},$$

shows that the last integral is

$$B(\mu, 1-\mu) = \Gamma(\mu)\Gamma(1-\mu) = \frac{\pi}{\sin \pi\mu}.$$