PROOF OF FORMULA 3.323.1

$$\int_1^\infty e^{-2ax-x^2}\,dx = \frac{\sqrt{\pi}}{2} e^{a^2}\left[1-\operatorname{erf}(1+a)\right] = \frac{\sqrt{\pi}}{2} e^{a^2}\operatorname{erfc}(1+a)$$

Complete the square to obtain

$$-x^2 - 2ax = -(x+a)^2 + a^2.$$

The change of variables $t = x + a$ gives

$$\int_1^\infty e^{-2ax-x^2}\,dx = e^{a^2}\int_{1+a}^\infty e^{-t^2}\,dt.$$

The result now follows from the representation

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt$$

(that appears as entry **8.250.1**) and its complementary value, sometimes denoted by erfc (that appears as entry **8.250.4**)

$$\operatorname{erfc}(x) = 1 - \operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}dt.$$