

PROOF OF FORMULA 3.361.1

$$\int_0^a \frac{e^{-qx}}{\sqrt{x}} dx = \sqrt{\frac{\pi}{q}} \operatorname{erf}(\sqrt{aq})$$

Let $x = t^2$ to obtain

$$\int_0^a \frac{e^{-qx}}{\sqrt{x}} dx = 2 \int_0^{\sqrt{a}} e^{-qt^2} dt.$$

The change of variables $t = v/\sqrt{q}$ gives

$$2 \int_0^{\sqrt{a}} e^{-qt^2} dt = \frac{2}{\sqrt{q}} \int_0^{\sqrt{qa}} e^{-v^2} dv.$$

The result follows now from the definition of the *error function*

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$$