# PROOF OF FORMULA 3.364.3

$$\int_0^\infty \frac{e^{-px}\,dx}{\sqrt{x(x+a)}} = e^{ap/2} K_0\left(\frac{ap}{2}\right)$$

The change of variable $x = ta$ yields

$$\int_0^\infty \frac{e^{-px}\,dx}{\sqrt{x(x+a)}} = \int_0^\infty \frac{e^{-pat}\,dt}{\sqrt{t(1+t)}}.$$

Now let $\sigma = 2t$ and observe that $t^2 + t = \frac{1}{4}\left[(\sigma+1)^2 - 1\right]$, to obtain

$$\int_0^\infty \frac{e^{-pat}\,dt}{\sqrt{t(1+t)}} = \int_0^\infty \frac{e^{-pa\sigma/2}\,d\sigma}{\sqrt{(\sigma+1)^2-1}}.$$

The change of variabel $u = \sigma + 1$ produces

$$\int_0^\infty \frac{e^{-pa\sigma/2}\,d\sigma}{\sqrt{(\sigma+1)^2-1}} = e^{pa/2} \int_1^\infty \frac{e^{-pau/2}\,du}{\sqrt{u^2-1}}.$$

The integral representation

$$K_\nu(z) = \frac{\Gamma(\frac{1}{2})}{\Gamma(\nu+\frac{1}{2})} \left(\frac{z}{2}\right)^\nu \int_1^\infty e^{-zt}(t^2-1)^{\nu-\frac{1}{2}}\,dt,$$

that, for $\nu = 0$ becomes

$$K_0(z) = \int_1^\infty \frac{e^{-zt}\,dt}{\sqrt{t^2-1)}}$$

gives the result.