FORMULA 3.381.2

$$\int_0^u x^{p-1}e^{-x}\,dx = \sum_{k=0}^{\infty}(-1)^k \frac{u^{p+k}}{k!(p+k)}$$
$$= e^{-u}\sum_{k=0}^{\infty}\frac{u^{p+k}}{p(p+1)\cdots(p+k)}$$

Expanding the integrand yields

$$\int_0^u x^{p-1}e^{-x}\,dx = \sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\int_0^u x^{p+k-1}\,dx$$
$$= \sum_{k=0}^{\infty}(-1)^k \frac{u^{p+k}}{k!(p+k)}.$$

To check the second identity, let $t = u - x$ to obtain

$$\int_0^u x^{p-1}e^{-x}\,dx = e^{-u}\int_0^u (u-t)^{p-1}e^{-t}\,dt$$
$$= e^{-u}\sum_{k=0}^{\infty}\frac{1}{k!}\int_0^u t^k(u-t)^{p-1}\,dt.$$

The inner integral is simplified via $t = us$ to get

$$\int_0^u t^k(u-t)^{p-1}\,dt = u^{p+k}\int_0^1 s^k(1-s)^{p-1}\,ds.$$

The integral is identified as $B(k+1,p)$ and the result follows from

$$B(k+1,p) = \frac{\Gamma(k+1)\Gamma(p)}{\Gamma(k+1+p)} = \frac{k!(p-1)!}{(k+p)!}.$$