PROOF OF FORMULA 3.411.15

$$\int_0^\infty \frac{x^2 e^{-nx}\,dx}{1+e^{-x}} = 2\sum_{k=n}^\infty \frac{(-1)^{n+k}}{k^3} = (-1)^{n+1}\left(\frac{3}{2}\zeta(3) + 2\sum_{k=1}^{n-1}\frac{(-1)^k}{k^3}\right)$$

Write

$$\int_0^\infty \frac{x^2 e^{-nx}}{1+e^{-x}}\,dx = \int_0^\infty \frac{x^2 e^{-(n-1)x}}{1+e^{x}}\,dx,$$

and use **3.411.8**

$$\int_0^\infty \frac{x^{m-1} e^{-px}}{1+e^{x}}\,dx = (m-1)!\sum_{k=1}^\infty \frac{(-1)^{k-1}}{(k+p)^m}$$

to obtain

$$\int_0^\infty \frac{x^2 e^{-nx}}{1+e^{-x}}\,dx = 2\sum_{k=1}^\infty \frac{(-1)^{k-1}}{(k+n-1)^3} = 2\sum_{k=n}^\infty \frac{(-1)^{k+n}}{k^3}.$$

The result is simplified using

$$\sum_{k=1}^\infty \frac{(-1)^k}{k^3} = -\frac{3}{4}\sum_{k=1}^\infty \frac{1}{k^3},$$

that is obtained by the usual even-odd splitting trick.