PROOF OF FORMULA 3.419.5

$$\int_{-\infty}^{\infty} \frac{x^4\,dx}{(\beta+e^x)(1-e^{-x})} = \frac{\left(\pi^2+\ln^2\beta\right)\left(7\pi^2+3\ln^2\beta\right)\,\ln\beta}{15(\beta+1)}$$

In **Part 1**, it has been shown that

$$h_n(a) = \int_0^\infty \frac{\ln^{n-1} t\,dt}{(t-1)(t+a)}$$

is given by

$$\begin{aligned} n(1+a)h_n(a) &= (-1)^n n!\,[1+(-1)^n]\,\zeta(n) \\ &+ \sum_{j=0}^{\lfloor n/2 \rfloor} \binom{n}{2j} \left(2^{2j}-2\right)(-1)^{j-1} B_{2j}\pi^{2j} \ln^{n-2j} a. \end{aligned}$$

The change of variables $t = e^{-x}$ shows that

$$h_n(a) = \int_{-\infty}^{\infty} \frac{x^{n-1}\,dx}{(1-e^{-x})(a+e^{-x})}.$$

The present entry corresponds to the value $n=5$.