PROOF OF FORMULA 3.434.2

$$\int_0^\infty \frac{e^{-\mu x} - e^{-\nu x}}{x}\, dx = \ln \frac{\nu}{\mu}$$

Define

$$f(\mu) := \int_0^\infty \frac{e^{-\mu x} - e^{-\nu x}}{x}\, dx.$$

Then $f(\nu) = 0$ and

$$f'(\mu) = -\int_0^\infty e^{-\mu x}\, dx = -\frac{1}{\mu}.$$

It follows that $f(\mu) = -\ln \mu + C$. The constant of integration C is determined to be $\ln \nu$ from $f(\nu) = 0$.