PROOF OF FORMULA 3.435.2

$$\int_0^\infty \frac{1-e^{-\mu x}}{x(x+b)}\,dx = \frac{1}{b}\left[\ln(b\mu)+\gamma - e^{b\mu}\text{Ei}(-b\mu)\right]$$

The change of variables $x = bt$ yields

$$\int_0^\infty \frac{1-e^{-\mu x}}{x(x+b)}\,dx = \frac{1}{b}\int_0^\infty \frac{1-e^{-at}}{t(1+t)}\,dt$$

with $a = b\mu$. This can be written as

$$\int_0^\infty \frac{1-e^{-at}}{t(1+t)}\,dt = \int_0^\infty \left(\frac{1}{1+t} - e^{-t}\right)\frac{dt}{t} + \int_0^\infty \left(\frac{e^{-t}}{t} - \frac{e^{-at}}{t(1+t)}\right)\,dt.$$

Entry **3.435.3** states that the first integral is γ. Using the partial fraction decomposition

$$\frac{1}{t(1+t)} = \frac{1}{t} - \frac{1}{1+t}$$

we obtain

$$\int_0^\infty \frac{1-e^{-at}}{t(1+t)}\,dt = \gamma + \int_0^\infty \frac{e^{-t}-e^{-at}}{t}\,dt + \int_0^\infty \frac{e^{-at}}{1+t}\,dt.$$

Entry **3.434.2** states that the first integral is $\ln a$. The change of variables $s = (1+t)/a$ shows that the last integral is $-e^{a}\text{Ei}(-a)$.