

PROOF OF FORMULA 3.435.3

$$\int_0^{\infty} \left(\frac{1}{1+x} - e^{-x} \right) \frac{dx}{x} = \gamma$$

The integral representation for the digamma function

$$\psi(a) = \int_0^{\infty} \left(e^{-x} - \frac{1}{(1+x)^a} \right) \frac{dx}{x},$$

appears in **8.361.2**. The special case $a = 1$ and the value $\psi(1) = -\gamma$ give the result.