PROOF OF FORMULA 3.462.6

$$\int_{-\infty}^{\infty} xe^{-px^2+2qx}\,dx = \frac{q}{p}\sqrt{\frac{\pi}{p}}\,e^{q^2/p}$$

Complete the square gives

$$-px^2 + 2qx = -p\left[(x - p/q)^2 - q^2/p\right].$$

The change of variables $t = x - q/p$ gives

$$\int_{-\infty}^{\infty} xe^{-px^2+2qx}\,dx = e^{q^2/p}\int_{-\infty}^{\infty}(t + q/p)e^{-pt^2}\,dt.$$

The integral

$$\int_{-\infty}^{\infty} te^{-pt^2}\,dt = 0$$

by symmetry. The second one can be scaled to get

$$\int_{-\infty}^{\infty} e^{-pt^2}\,dt = \frac{1}{\sqrt{p}}\int_{-\infty}^{\infty} e^{-y^2}\,dy = \sqrt{\frac{\pi}{p}}.$$

This is the result.