PROOF OF FORMULA 3.522.10

$$\int_0^\infty \frac{x\,dx}{(1+x^2)\,\cosh\frac{\pi x}{4}} = \frac{1}{\sqrt{2}}\left(\pi - 2\ln(\sqrt{2}+1)\right)$$

This is entry **3.522.3** with $a = \pi/4$ and $b = 1$. Therefore

$$\int_0^\infty \frac{x\,dx}{(1+x^2)\,\cosh\frac{\pi x}{4}} = 4\sum_{k=1}^\infty \frac{(-1)^{k-1}}{4k-1}.$$

To evaluate the series integrate

$$\sum_{k=1}^\infty (-1)^k x^{4k-2} = \frac{x^2}{1+x^4}$$

to obtain

$$\sum_{k=1}^\infty \frac{(-1)^{k-1}}{4k-1} = \int_0^1 \frac{x^2\,dx}{1+x^4}.$$

The factorization

$$1+x^4 = (x^2-\sqrt{2}x+1)(x^2+\sqrt{2}x+1)$$

gives the integral by partial fractions.