PROOF OF FORMULA 3.522.5

$$\int_0^\infty \frac{x\,dx}{(1+x^2)\sinh \pi x} = \ln 2 - \frac{1}{2}$$

This is the special case $b = 1$ of entry **3.522.2** that gives

$$\int_0^\infty \frac{x\,dx}{(1+x^2)\sinh \pi x} = \frac{1}{2} - \beta(2),$$

where

$$\beta(2) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k+2} = \sum_{k=2}^{\infty} \frac{(-1)^k}{k} = -\ln 2 + 1.$$

This gives the result.