PROOF OF FORMULA 3.522.9

$$\int_0^\infty \frac{x\,dx}{(1+x^2)\sinh\frac{\pi x}{4}} = \frac{1}{\sqrt{2}}\Big(\pi + 2\ln(\sqrt{2}+1)\Big) - 2$$

This is entry **3.522.1** with $a = \pi/4$ and $b = 1$. Therefore

$$\int_0^\infty \frac{x\,dx}{(1+x^2)\sinh\frac{\pi x}{4}} = 2 + 4\sum_{k=1}^\infty \frac{(-1)^k}{4k+1}.$$

To evaluate the series integrate

$$\sum_{k=1}^\infty (-1)^k x^{4k} = -\frac{x^4}{1+x^4}$$

to obtain

$$\sum_{k=1}^\infty \frac{(-1)^k}{4k+1} = -1 + \int_0^1 \frac{dx}{1+x^4}.$$

The factorization

$$1 + x^4 = (x^2 - \sqrt{2}x + 1)(x^2 + \sqrt{2}x + 1)$$

gives the integral by partial fractions.