# PROOF OF FORMULA 3.523.3

$$\int_0^\infty \frac{x^{b-1}\,dx}{\cosh ax} = \frac{2\Gamma(b)}{(2a)^b} \sum_{k=0}^\infty (-1)^k \left( \frac{2}{2k+1} \right)^b$$

The change of variable $t = ax$ gives

$$\int_0^\infty \frac{x^{b-1}\,dx}{\cosh ax} = \frac{2}{a^b} \int_0^\infty \frac{t^{b-1}\,e^{-t}\,dt}{1+e^{-2t}}.$$

Expand the integrand in series to obtain

$$\int_0^\infty \frac{t^{b-1}\,e^{-t}\,dt}{1+e^{-2t}} = \sum_{k=0}^\infty (-1)^k \int_0^\infty t^{b-1} e^{-(2k+1)t}\,dt.$$

The change of variables $u = (2k+1)t$ gives the result after recognizing the value

$$\int_0^\infty u^{b-1} e^{-u}\,du = \Gamma(b).$$