PROOF OF FORMULA 3.527.11

$$\int_0^\infty \frac{x \sinh ax}{(\cosh x)^{2\mu+1}}\,dx = \frac{\sqrt{\pi}}{4\mu a^2}\frac{\Gamma(\mu)}{\Gamma(\mu+\frac{1}{2})}$$

Differentiate entry **3.512.1**

$$\int_0^\infty \frac{dx}{(\cosh ax)^{2\mu}} = \frac{4^{\mu-1}}{a}B(\mu,\mu)$$

with respect to μ. This yields

$$\int_0^\infty \frac{x \sinh ax}{(\cosh x)^{2\mu+1}}\,dx = \frac{2^{2\mu-2}B(\mu,\mu)}{\mu a^2}.$$

To obtain the form of the answer given here, use the duplication formula for the gamma function

$$\Gamma(2\mu) = \frac{2^{2\mu-1}}{\sqrt{\pi}}\Gamma(\mu)\Gamma(\mu+\tfrac{1}{2})$$

to establish

$$B(\mu,\mu) = 2^{1-2\mu}B(\tfrac{1}{2},\mu).$$

Then replace.