# PROOF OF FORMULA 3.541.2

$$\int_0^\infty e^{-\mu x} \frac{\sinh \beta x}{\sinh bx} \, dx = \frac{1}{2b} \left[ \psi \left( \frac{1}{2} + \frac{\mu+\beta}{2b} \right) - \psi \left( \frac{1}{2} + \frac{\mu-\beta}{2b} \right) \right]$$

Write the integral as

$$\begin{aligned} \int_0^\infty e^{-\mu x} \frac{\sinh \beta x}{\sinh bx} \, dx &= \int_0^\infty \frac{e^{-(\mu-\beta)x} - e^{-(\mu+\beta)x}}{e^x - e^{-x}} \\ &= \int_0^\infty \frac{e^{-(\mu-\beta+1)x} - e^{-(\mu+\beta+1)x}}{1 - e^{-2x}} \\ &= \frac{1}{2} \int_0^\infty \frac{e^{-(\mu-\beta+1)s/2} - e^{-(\mu+\beta+1)s/2}}{1 - e^{-s}} \, ds, \end{aligned}$$

where we have used $s = 2x$ in the last step.

The result now follows from entry **8.361.5**

$$\psi(z) = \int_0^\infty \frac{e^{-t} - e^{-zt}}{1 - e^{-t}} \, dt - \gamma.$$