

PROOF OF FORMULA 3.541.4

$$\int_0^{\infty} e^{-x} \frac{\sinh ax}{\sinh x} dx = \frac{1}{a} - \frac{\pi}{2} \cot\left(\frac{\pi a}{2}\right)$$

This is the special case $\mu = 1$, $\beta = a$, $b = 1$ of entry **3.541.2** that gives the evaluation

$$\int_0^{\infty} e^{-x} \frac{\sinh ax}{\sinh x} dx = \frac{1}{2} \left[\psi\left(1 + \frac{a}{2}\right) - \psi\left(1 - \frac{a}{2}\right) \right].$$

This expression can be reduced to the stated one by employing

$$\psi(x+1) = \psi(x) + \frac{1}{x}$$

and

$$\psi(1-x) = \psi(x) + \pi \cot \pi x.$$