PROOF OF FORMULA 3.541.9

$$\int_0^\infty e^{-\mu x} \frac{\sinh \mu x}{\cosh^2 \mu x} \, dx = \frac{1 - \ln 2}{\mu}$$

The change of variables $t = \mu x$ shows that it suffices to assume $\mu = 1$. Therefore we prove

$$\int_0^\infty e^{-t} \frac{\sinh t}{\cosh^2 t} \, dt = 1 - \ln 2.$$

Write the integral as

$$\int_0^\infty e^{-t} \frac{\sinh t}{\cosh^2 t} \, dt = 2 \int_0^\infty \frac{e^{-2t}(1 - e^{-2t})}{(1 + e^{-2t})^2} \, dt$$

and make the change of variables $u = 1 + e^{-2t}$ to obtain

$$\int_0^\infty e^{-t} \frac{\sinh t}{\cosh^2 t} \, dt = - \int_1^2 \left(\frac{1}{u} - \frac{2}{u^2} \right) \, du.$$

This last integral is $1 - \ln 2$ as claimed.