PROOF OF FORMULA 3.562.3

$$\int_0^\infty xe^{-\beta x^2}\sinh\gamma x\,dx = \frac{\gamma}{4\beta}\sqrt{\frac{\pi}{\beta}}e^{\gamma^2/4\beta}$$

Write the integral as

$$\begin{aligned}
\int_0^\infty xe^{-\beta x^2}\sinh\gamma x\,dx &= \frac{1}{2}\int_0^\infty xe^{-\beta x^2+\gamma x}\,dx - \frac{1}{2}\int_0^\infty xe^{-\beta x^2-\gamma x}\,dx \\
&= \frac{1}{2}e^{\gamma^2/4\beta}\int_{-\gamma/2\beta}^\infty \left(t+\frac{\gamma}{2\beta}\right)e^{-\beta t^2}\,dt - \frac{1}{2}e^{\gamma^2/4\beta}\int_{\gamma/2\beta}^\infty \left(t-\frac{\gamma}{2\beta}\right)e^{-\beta t^2}\,dt \\
&= \frac{1}{2}e^{\gamma^2/4\beta}\int_{-\gamma/2\beta}^\infty te^{-\beta t^2}\,dt + \frac{\gamma}{4\beta}e^{\gamma^2/4\beta}\int_{-\gamma/2\beta}^\infty e^{-\beta t^2}\,dt \\
&\quad -\frac{1}{2}e^{\gamma^2/4\beta}\int_{\gamma/2\beta}^\infty te^{-\beta t^2}\,dt + \frac{\gamma}{4\beta}e^{\gamma^2/4\beta}\int_{\gamma/2\beta}^\infty e^{-\beta t^2}\,dt.
\end{aligned}$$

The first and third integral cancel each other because the integrand is odd so the part between $-\gamma/2\beta$ and $\gamma/2\beta$ vanishes. The second and fourth give

$$\begin{aligned}
\int_0^\infty xe^{-\beta x^2}\sinh\gamma x\,dx &= \frac{\gamma}{4\beta}e^{\gamma^2/4\beta}\left(\int_{-\gamma/2\beta}^\infty e^{-\beta t^2}\,dt + \int_{\gamma/2\beta}^\infty e^{-\beta t^2}\,dt\right) \\
&= \frac{\gamma}{4\beta}e^{\gamma^2/4\beta}\int_{-\infty}^\infty e^{-\beta t^2}\,dt \\
&= \frac{\gamma}{4\beta}\sqrt{\frac{\pi}{\beta}}e^{\gamma^2/4\beta},
\end{aligned}$$

as claimed.