PROOF OF FORMULA 3.621.4

$$\int_0^{\pi/2} \sin^{2m+1} x \, dx = \int_0^{\pi/2} \cos^{2m+1} x \, dx = \frac{(2m)!!}{(2m+1)!!}$$

Let $t = \sin x$ to obtain

$$\int_0^{\pi/2} \sin^{2m+1} x \, dx = \int_0^1 t^{2m+1}(1-t^2)^{-1/2} \, dt.$$

The change of variables $s = t^2$ gives

$$\int_0^{\pi/2} \sin^{2m+1} x \, dx = \frac{1}{2} \int_0^1 s^m (1-s)^{-1/2} \, ds = \frac{1}{2} B\left(m+1, \tfrac{1}{2}\right).$$

Therefore

$$\int_0^{\pi/2} \sin^{2m+1} x \, dx = \frac{\Gamma(m+1)\Gamma(1/2)}{2\Gamma(m+3/2)}.$$

The result now follows from

$$\Gamma(m) = (m-1)! \text{ and } \Gamma(m+\tfrac{1}{2}) = \frac{\sqrt{\pi}}{2^m}(2m-1)!!,$$

followed by elementary simplifications.