PROOF OF FORMULA 3.623.3

$$\int_0^{\pi/4} \tan^\mu x \cos^2 x \, dx = \frac{(1-\mu)}{4} \beta\left(\frac{1+\mu}{2}\right) + \frac{1}{4}$$

Let $y = \tan x$ to produce

$$\int_0^{\pi/4} \tan^\mu x \cos^2 x \, dx = \int_0^1 \frac{y^\mu \, dy}{(1+y^2)^2}.$$

Entry **3.251.7** shows that

$$\int_0^1 \frac{y^\mu \, dy}{(1+y^2)^2} = -\frac{1}{4} + \frac{(\mu-1)}{4} \beta\left(\frac{\mu-1}{2}\right).$$

The formula is therefore equivalent to the identity

$$\frac{1-\mu}{4} \beta\left(\frac{\mu+1}{2}\right) + \frac{1}{4} = \frac{\mu-1}{4} \beta\left(\frac{\mu-1}{2}\right) - \frac{1}{4},$$

that reduces to

$$\beta\left(\frac{\mu+1}{2}\right) + \beta\left(\frac{\mu-1}{2}\right) = \frac{2}{\mu-1}.$$

This follows directly from the representation

$$\beta(a) = \int_0^1 \frac{t^{a-1} \, dt}{1+t}.$$