PROOF OF FORMULA 3.624.1

$$\int_0^{\pi/4} \frac{\sin^p x\, dx}{\cos^{p+2} x} = \frac{1}{p+1}$$

Start with

$$\begin{aligned} \int_0^{\pi/4} \frac{\sin^p x\, dx}{\cos^{p+2} x} &= \int_0^{\pi/4} \tan^p x \sec^2 \, dx \\ &= \int_0^{\pi/4} \tan^{p+2} x\, dx + \int_0^{\pi/4} \tan^p x\, dx. \end{aligned}$$

Formula **3.622.2** shows that

$$\int_0^{\pi/4} \tan^a x\, dx = \frac{1}{2}\beta\left(\frac{a+1}{2}\right),$$

therefore

$$\int_0^{\pi/4} \frac{\sin^p x\, dx}{\cos^{p+2} x} = \frac{1}{2}\beta\left(\frac{p+3}{2}\right) + \frac{1}{2}\beta\left(\frac{p+1}{2}\right).$$

The identity

$$\beta(a) + \beta(a+1) = \frac{1}{a}$$

that follows directly from

$$\beta(a) = \frac{1}{2}\left[\psi\left(\frac{a+1}{2}\right) - \psi\left(\frac{a}{2}\right)\right],$$

gives the result.