

PROOF OF FORMULA 3.624.2

$$\int_0^{\pi/2} \frac{\sin^{\mu-\frac{1}{2}} x \, dx}{\cos^{2\mu-1} x} = \int_0^{\pi/2} \frac{\cos^{\mu-\frac{1}{2}} x \, dx}{\sin^{2\mu-1} x} = \frac{\Gamma\left(\frac{\mu}{2} + \frac{1}{4}\right) \Gamma(1-\mu)}{2\Gamma\left(\frac{5}{4} - \frac{\mu}{2}\right)}$$

The integral representation

$$B(u, v) = 2 \int_0^{\pi/2} \sin^{2u-1} x \cos^{2v-1} x \, dx,$$

shows that

$$\int_0^{\pi/2} \frac{\sin^{\mu-\frac{1}{2}} x \, dx}{\cos^{2\mu-1} x} = \frac{1}{2} B\left(\frac{\mu}{2} + \frac{1}{4}, 1 - \mu\right).$$

The result now follows from the basic identity

$$B(u, v) = \frac{\Gamma(u) \Gamma(v)}{\Gamma(u+v)}.$$