PROOF OF FORMULA 3.624.4

$$\int_0^{\pi/4} \frac{\cos^\mu(2x)}{(\cos x)^{2(\mu+1)}} \, dx = 2^{2\mu} B(\mu+1, \mu+1)$$

The integral can be written as

$$\int_0^{\pi/4} \frac{\cos^\mu(2x)}{(\cos x)^{2(\mu+1)}} \, dx = 2^{\mu+1} \int_0^{\pi/4} \frac{(\cos 2x)^\mu \, dx}{(1+\cos 2x)^{\mu+1}}.$$

The change of variables $t = \cos 2x$ gives

$$\int_0^{\pi/4} \frac{\cos^\mu(2x)}{(\cos x)^{2(\mu+1)}} \, dx = 2^\mu \int_0^1 t^\mu (1-t)^{-1/2} (1+t)^{-\mu-3/2} \, dt.$$

An integral representation of the hypergeometric function is

$${}_2F_1\left[\alpha, \beta; \gamma; z\right] = \frac{1}{B(\beta, \gamma-\beta)} \int_0^1 t^{\beta-1} (1-t)^{\gamma-\beta-1} (1-tz)^{-\alpha} \, dt$$

shows that the original integral is

$$\int_0^{\pi/4} \frac{\cos^\mu(2x)}{(\cos x)^{2(\mu+1)}} \, dx = 2^\mu B(\mu+1, \tfrac{1}{2}) {}_2F_1\left[\mu+\tfrac{3}{2}, \mu+1; \mu+\tfrac{3}{2}; -1\right].$$

The result now follows from the specialization

$${}_2F_1\left[\alpha, \beta; \alpha; z\right] = (1-z)^{-\beta}.$$