PROOF OF FORMULA 3.625.4

$$\int_0^{\pi/4} \frac{\sin^{2n} x \, \cos^{m-1/2} 2x}{\cos^{2n+2m+1} x} \, dx = \frac{(2n-1)!! \, (2m-1)!!}{(2n+2m)!!} \frac{\pi}{2}$$

The change of variable $t = \tan x$ gives

$$\int_0^{\pi/4} \frac{\sin^{2n} x \, \cos^{m-1/2} 2x}{\cos^{2n+2m+1} x} \, dx = \int_0^1 t^{2n}(1-t^2)^{m-1/2} \, dt.$$

Now let $v = t^2$ and use the integral representation of the beta function

$$B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1} \, dt$$

to obtain

$$\int_0^{\pi/4} \frac{\sin^{2n} x \, \cos^{m-1/2} 2x}{\cos^{2n+2m+1} x} \, dx = \frac{1}{2} B\left(n + \tfrac{1}{2}, m + \tfrac{1}{2}\right).$$

This is now reduced to the stated formula using

$$\Gamma\left(p + \tfrac{1}{2}\right) = \frac{\sqrt{\pi}}{2^p}(2p-1)!!.$$