PROOF OF FORMULA 3.626.1

$$\int_0^{\pi/4} \frac{\sin^{2n-1} x}{\cos^{2n+2} x} \sqrt{\cos 2x}\, dx = \frac{(2n-2)!!}{(2n+1)!!}$$

Let $u = \tan x$ use $\cos x = 1/\sqrt{1+u^2}$, $\cos 2x = (1-u^2)/(1+u^2)$ and $dx = du/(1+u^2)$. Then use $v = t^2$ to obtain

$$\int_0^{\pi/4} \frac{\sin^{2n-1} x}{\cos^{2n+2} x} \sqrt{\cos 2x}\, dx = \frac{1}{2} \int_0^1 v^{n-1} (1-v)^{1/2}\, dv = \frac{1}{2} B\left(n, \frac{3}{2}\right).$$

The result now follows from

$$B(a,b) = \frac{\Gamma(a)\,\Gamma(b)}{\Gamma(a+b)}$$

and

$$\Gamma\left(n + \tfrac{1}{2}\right) = \frac{(2n-1)!!\,\sqrt{\pi}}{2^n}.$$