PROOF OF FORMULA 3.627

$$\int_0^{\pi/2} \frac{\tan^\mu x}{\cos^\mu x}\, dx = \int_0^{\pi/2} \frac{\cot^\mu x}{\sin^\mu x}\, dx = \frac{\Gamma(\mu)\Gamma\left(\frac{1}{2}-\mu\right)}{2^\mu\sqrt{\pi}} \sin\frac{\pi\mu}{2}$$

The equality among the two integrals follows from the change of variables $x \mapsto \pi/2 - x$. The first integral is

$$\int_0^{\pi/2} \frac{\tan^\mu x}{\cos^\mu x}\, dx = \int_0^{\pi/2} \sin^\mu x \cos^{-2\mu} x\, dx$$

and the can be expressed in terms of the beta function as

$$\int_0^{\pi/2} \sin^\mu x \cos^{-2\mu} x\, dx = \frac{1}{2} B\left(\frac{\mu+1}{2}, \frac{1}{2}-\mu\right) = \frac{\Gamma\left(\frac{1}{2}+\frac{\mu}{2}\right)\Gamma\left(\frac{1}{2}-\mu\right)}{2\Gamma\left(1-\frac{\mu}{2}\right)}.$$

The identities

$$\Gamma\left(x+\tfrac{1}{2}\right) = \frac{\Gamma(2x)\sqrt{\pi}}{\Gamma(x)2^{2x-1}}$$

and

$$\Gamma(x)\Gamma(1-x) = \frac{\pi}{\sin \pi x}$$

give the result.