

PROOF OF FORMULA 3.628

$$\int_0^{\pi/2} \sec^{2p} x \sin^{2p-1} x \, dx = \frac{1}{2\sqrt{\pi}} \Gamma(p) \Gamma\left(\frac{1}{2} - p\right)$$

The integral representation

$$B(a, b) = 2 \int_0^{\pi/2} \sin^{2a-1} x \cos^{2b-1} x \, dx$$

gives the value

$$\int_0^{\pi/2} \sec^{2p} x \sin^{2p-1} x \, dx = \frac{1}{2} B\left(p, \frac{1}{2} - p\right).$$

This reduces to the answer given here, using

$$B(a, b) = \frac{\Gamma(a) \Gamma(b)}{\Gamma(a+b)}.$$