PROOF OF FORMULA 3.636.2

$$\int_0^{\pi/2} (\tan x)^{\pm\mu} \cos 2x \, dx = \mp \frac{\pi\mu}{2} \sec \frac{\pi\mu}{2}$$

The case of the + sign is considered in detail, the other one is similar. Write the integral as

$$\begin{aligned} \int_0^{\pi/2} (\tan x)^{\mu} \cos 2x \, dx &= \int_0^{\pi/2} \sin^{\mu} x \cos^{2-\mu} x \, dx - \int_0^{\pi/2} \sin^{\mu+2} x \cos^{-\mu} x \, dx \\ &= \frac{1}{2} B\left(\frac{1+\mu}{2}, \frac{3-\mu}{2}\right) - \frac{1}{2} B\left(\frac{3+\mu}{2}, \frac{1-\mu}{2}\right). \end{aligned}$$

This reduces to

$$\int_0^{\pi/2} (\tan x)^{\mu} \cos 2x \, dx = -\frac{\mu}{2} \Gamma\left(\frac{1}{2} + \frac{\mu}{2}\right) \Gamma\left(\frac{1}{2} - \frac{\mu}{2}\right).$$

The result now follows from the identity

$$\Gamma(\tfrac{1}{2} + x)\Gamma(\tfrac{1}{2} - x) = \frac{\pi}{\cos \pi x}.$$