PROOF OF FORMULA 3.638.1

$$\int_0^{\pi/4} \frac{\sin^{2\mu} x\, dx}{(\cos 2x)^{\mu+1/2}\, \cos x} = \frac{\pi}{2} \sec \pi\mu$$

The change of variables $t = \tan x$ gives

$$\int_0^{\pi/4} \frac{\sin^{2\mu} x\, dx}{(\cos 2x)^{\mu+1/2}\, \cos x} = \int_0^1 t^{2\mu}(1-t^2)^{-\mu-1/2}\, dt.$$

Now let $t = v^2$ to obtain

$$\int_0^{\pi/4} \frac{\sin^{2\mu} x\, dx}{(\cos 2x)^{\mu+1/2}\, \cos x} = \frac{1}{2}\int_0^1 v^{\mu-1/2}(1-v)^{-\mu-1/2}\, dv.$$

This last integral is

$$B(\mu+\tfrac{1}{2}, -\mu+\tfrac{1}{2}) = \Gamma(\mu+\tfrac{1}{2})\Gamma(\tfrac{1}{2}-\mu) = \frac{\pi}{\cos \pi\mu}$$

and the result follows.