PROOF OF FORMULA 3.644.3

$$\int_0^\pi \frac{\sin^m x \, dx}{1 - \cos x} = 2^{m-1} B\left(\frac{m-1}{2}, \frac{m+1}{2}\right)$$

Write the integral as

$$\int_0^\pi \frac{\sin^m x \, dx}{1 - \cos x} = \int_0^\pi (1 + \cos x)^{m/2} (1 - \cos x)^{m/2 - 1} \, dx.$$

The change of variables $t = 1 - \cos x$ produces

$$
\begin{aligned}
\int_0^\pi \frac{\sin^m x \, dx}{1 - \cos x} &= \int_0^\pi (1 + \cos x)^{m/2} (1 - \cos x)^{m/2 - 1} \, dx \\
&= \int_0^2 t^{m/2 - 1} (2 - t)^{m/2} \frac{dt}{\sqrt{t(2 - t)}} \\
&= 2^{m-1} \int_0^1 s^{m/2 - 3/2} (1 - s)^{m/2 - 1/2} \, ds
\end{aligned}
$$

using $t = 2s$ in the last step. This is the result.