PROOF OF FORMULA 3.662.3

$$\int_0^{\pi/2} (\sec x - 1)^\mu \, \tan x \, dx = \int_0^{\pi/2} (\csc x - 1)^\mu \, \cot x \, dx = -\frac{\pi}{\sin \pi\mu}$$

The first integral is

$$\int_0^{\pi/2} (\sec x - 1)^\mu \, \tan x \, dx = \int_0^{\pi/2} \cos^{-1-\mu} x (1 - \cos x)^\mu \, \sin x \, dx.$$

The change of variables $t = \cos x$ gives

$$\int_0^{\pi/2} (\sec x - 1)^\mu \, \tan x \, dx = \int_0^1 t^{-1-\mu} (1 - t)^\mu \, dt.$$

This is evaluated as

$$B(-\mu, 1 + \mu) = \Gamma(-\mu)\Gamma(1 - \mu) = -\frac{\pi}{\sin \pi\mu}$$

as claimed. The second integral comes from the change of variable $x \mapsto \frac{\pi}{2} - x$.