PROOF OF FORMULA 3.691.3

$$\int_0^1 \cos(ax^2)\,dx = \sqrt{\frac{\pi}{2a}}\,C(\sqrt{a})$$

The *Fresnel cosine integral* C is defined by

$$C(x) := \sqrt{\frac{2}{\pi}}\int_0^x \cos(t^2)\,dt.$$

The change of variables $t = \sqrt{a}\,x$ gives

$$\int_0^1 \cos(ax^2)\,dx = \frac{1}{\sqrt{a}}\int_0^{\sqrt{a}} \cos(t^2)\,dt.$$

This yields the result.