

PROOF OF FORMULA 4.212.2

$$\int_0^1 \frac{dx}{a - \ln x} = -e^a \text{Ei}(-a)$$

Let $t = a - \ln x$ to obtain

$$\int_0^1 \frac{dx}{a - \ln x} = -e^a \int_{-\infty}^{-a} \frac{e^t}{t} dt.$$

The integral is the function $\text{Ei}(-a)$.