

PROOF OF FORMULA 4.212.5

$$\int_0^1 \frac{\ln x \, dx}{(a + \ln x)^2} = 1 + (1 - a)e^{-a}\text{Ei}(a)$$

Let $t = a + \ln x$ to obtain

$$\int_0^1 \frac{\ln x \, dx}{(a + \ln x)^2} = e^{-a} \left(\int_{-\infty}^a \frac{e^t}{t} dt - a \int_{-\infty}^a \frac{e^t}{t^2} dt \right).$$

The first integral is $\text{Ei}(a)$. The second integral follows from **2.325.2** that states

$$\int \frac{e^{ax}}{x^2} dx = -\frac{e^{ax}}{x} + a\text{Ei}(ax).$$