PROOF OF FORMULA 4.212.8

$$\int_0^1 \frac{dx}{(a+\ln x)^n} = \frac{e^{-a}\,\mathrm{Ei}(a)}{(n-1)!} - \frac{1}{(n-1)!}\sum_{k=1}^{n-1} \frac{(n-k-1)!}{a^{n-k}}$$

Let $t = a + \ln x$ to obtain

$$\int_0^1 \frac{dx}{(a+\ln x)^n} = e^{-a} \int_{-\infty}^{a} \frac{e^t}{t^n} dt.$$

The indefinite integral is evaluated in **2.324.2** as

$$\int \frac{e^t}{t^n} dt = -\frac{e^t}{(n-1)!} \sum_{k=1}^{n-1} \frac{(n-k-1)!}{x^{n-k}} + \frac{\mathrm{Ei}(t)}{(n-1)!}.$$

Now evaluate at $t = -\infty$ and $t = a$. Keep in mind that $\mathrm{Ei}(-\infty) = 0$.