PROOF OF FORMULA 4.212.9

$$\int_0^1 \frac{dx}{(a-\ln x)^n} = \frac{(-1)^n e^a \,\mathrm{Ei}(-a)}{(n-1)!} + \frac{(-1)^{n-1}}{(n-1)!} \sum_{k=1}^{n-1} \frac{(n-k-1)!}{(-a)^{n-k}}$$

Let $t = \ln x - a$ to obtain

$$\int_0^1 \frac{dx}{(a-\ln x)^n} = (-1)^n e^a \int_{-\infty}^{-a} \frac{e^t}{t^n} dt.$$

The indefinite integral is evaluated in **2.324.2** as

$$\int \frac{e^t}{t^n} dt = -\frac{e^t}{(n-1)!} \sum_{k=1}^{n-1} \frac{(n-k-1)!}{x^{n-k}} + \frac{\mathrm{Ei}(t)}{(n-1)!}.$$

Now evaluate at $t = -\infty$ and $t = -a$. Keep in mind that $\mathrm{Ei}(-\infty) = 0$.