PROOF OF FORMULA 4.224.2

$$\int_0^{\pi/4} \ln \sin x \, dx = -\frac{\pi}{4} \ln 2 - \frac{G}{2}$$

The Catalan constant has the integral representation

$$G = -\int_0^{\pi/4} \ln \tan x \, dx$$

given in **4.227.2**. This gives

$$\int_0^{\pi/4} \ln \sin x \, dx - \int_0^{\pi/4} \ln \cos x \, dx = -G.$$

On the other hand,

$$
\begin{aligned}
\int_0^{\pi/4} \ln \sin x \, dx + \int_0^{\pi/4} \ln \cos x \, dx &= \int_0^{\pi/4} \ln(\sin x \cos x) \, dx \\
&= \int_0^{\pi/4} \ln\left(\frac{\sin 2x}{2}\right) dx \\
&= \frac{1}{2} \int_0^{\pi/2} \ln \sin t \, dt - \frac{\pi}{4} \ln 2.
\end{aligned}
$$

Using the evaluation

$$\int_0^{\pi/2} \ln \sin t \, dt = -\frac{\pi}{2} \ln 2$$

given in **4.224.3** produces

$$\int_0^{\pi/4} \ln \sin x \, dx + \int_0^{\pi/4} \ln \cos x \, dx = -\frac{\pi}{2} \ln 2.$$

Solving the system of equation for the two unknown integrals gives

$$\int_0^{\pi/4} \ln \sin x \, dx = -\frac{\pi}{4} \ln 2 - \frac{G}{2},$$

and

$$\int_0^{\pi/4} \ln \cos \, dx = -\frac{\pi}{4} \ln 2 + \frac{G}{2},$$