

**PROOF OF FORMULA 4.227.15**

$$\int_0^{\pi/4} \ln(\tan x + \cot x) dx = \frac{1}{2} \int_0^{\pi/2} \ln(\tan x + \cot x) dx = \frac{\pi}{2} \ln 2$$

The change of variables  $t = \pi/2 - x$  proves the first equality. To obtain the value of the integral, write it as

$$\int_0^{\pi/2} \ln(\tan x + \cot x) dx = - \int_0^{\pi/2} \ln\left(\frac{\sin 2x}{2}\right) dx.$$

The change of variables  $t = 2x$  gives

$$\int_0^{\pi/2} \ln(\tan x + \cot x) dx = \frac{\pi}{2} \ln 2 - \frac{1}{2} \int_0^{\pi} \ln \sin t dt.$$

Symmetry now gives

$$\int_0^{\pi/2} \ln(\tan x + \cot x) dx = \frac{\pi}{2} \ln 2 - \int_0^{\pi/2} \ln \sin t dt$$

and the result follows from entry **4.224.3**

$$\int_0^{\pi/2} \ln \sin t dt = -\frac{\pi}{2} \ln 2.$$