

PROOF OF FORMULA 4.227.2

$$\int_0^{\pi/4} \ln \tan x \, dx = - \int_{\pi/4}^{\pi/2} \ln \tan x \, dx = -G$$

The Catalan constant has the integral representation

$$G = - \int_0^1 \frac{\ln x \, dx}{1+x^2}.$$

This appears as **4.231.12**. The change of variables $x = \tan t$ gives the first evaluation. The change of variables $x \mapsto \pi/2 - x$ shows that

$$\int_0^{\pi/2} \ln \tan x \, dx = 0.$$

This gives the second evaluation.