FORMULA 4.227.5

$$\int_0^{\pi/2} \ln^{2n} \tan x \, dx \;=\; 2(2n)! \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^{2n+1}}$$

$$=\; \left(\frac{\pi}{2}\right)^{2n+1} |E_{2n}| \text{ if } n \text{ is even}$$

Observe that

$$\int_{\pi/4}^{\pi/2} \ln^{2n} \tan x \, dx = \int_0^{\pi/4} \ln^{2n} \cot x \, dx = \int_0^{\pi/4} \ln^{2n} \tan x \, dx$$

and now the result follows from entry **4.227.4**.