PROOF OF FORMULA 4.229.3

$$\int_0^1 \ln(\ln 1/x)\frac{dx}{\sqrt{\ln 1/x}} = -(\gamma + 2\ln 2)\sqrt{\pi}$$

Entry **4.229.4** states that

$$\int_0^1 \ln(\ln 1/x)\,(\ln 1/x)^{\mu-1}\,dx = \psi(\mu)\Gamma(\mu)$$

The result now follows from the values $\psi(1/2) = -(\gamma + 2\ln 2)$ (given as entry **8.366.2** and $\Gamma(1/2) = \sqrt{\pi}$.